The month was unusually cold, cloudy, and stormy, and the advance of vegetation, as well as the progress of farm work incident to the season, was greatly delayed. Snow fell to unusual amounts in portions of the Great Valley division, and light to killing frosts occurred, but no special damage resulted from either.—*Edward A. Evans.*

Washington.—The mean temperature was 45.9°, or 2.2° below normal; the highest was 79°, at Pasco on the 10th, and the lowest, 11°, at Republic on the 3d. The average precipitation was 3.49, or 0.76 above normal; the greatest monthly amount, 13.42, occurred at Clearwater, and the least, trace, at Pasco.

The weather was in general too cool for rapid growth of crops, but was not unfavorable to winter wheat and early spring wheat. Spring seeding was late and fruit bloomed two or three weeks later than usual.—*G. N. Salisbury.*

West Virginia.—The mean temperature was 47.5°, or 5.1° below normal; the highest was 95°, at Point Pleasant on the 30th, and the lowest, 18°, at Philippi on the 1st. The average precipitation was 7.05, or 3.53 above normal; the greatest monthly amount, 10.70, occurred at Clay, and the least, 4.95, at Beverly.

The cold, stormy, and unseasonable weather, with excessive precipitation, was very unfavorable for farm work and the growth of vegetation, so that little advancement was made. At the close of the month work was behind, grass short, gardens backward, wheat below average condition, feed scarce, stock in poor condition and the prospects for fruit promising.—*E. C. Vose.*

Wisconsin.—The mean temperature was 46.7°, or 2.2° above normal; the highest was 90°, at Prairie du Chien and Pine River on the 30th, and the lowest, 12°, at Amherst on the 1st and at Spooner on the 19th. The average precipitation was 0.85, or 1.92 below normal; the greatest monthly amount, 2.22, occurred at Barron, and the least, trace, at West Bend.

The month was one of the driest Aprils on record, especially in the southern section, where the total precipitation was only about 20 per cent of the normal. The effect of the drought is most noticeable on meadows and pastures. Early sown grain is coming up nicely and preparations for corn and potatoes are progressing rapidly.—*W. M. Wilson.*

Wyoming.—The mean temperature was 40.3°, or 1.0° below normal; the highest was 88°, at Alcova on the 28th, and the lowest, 15° below zero, at Centennial on the 17th. The average precipitation was 1.31, or 0.40 below normal; the greatest monthly amount, 3.11, occurred at Lander, while none fell at Lovell (Byron P. O.)—*W. S. Palmer.*

SPECIAL CONTRIBUTIONS.

THE THEORY OF THE FORMATION OF PRECIPITATION ON MOUNTAIN SLOPES.

By Prof. F. POCKELS, School of Technology, Dresden, Germany. Translated from Ann. d. Physik, 1901. (4) Vol. III, pp. 459-480.

It is a well known principle of climatology that the side of a mountain range which is turned toward the prevailing wind has in general a greater precipitation than the plain on the windward side, and a still greater in comparison with the leeward side of the mountain range. There has been no doubt as to the explanation of this phenomenon since it has been recognized that the principal cause of the condensation of the aqueous vapor is the adiabatic cooling of the rising mass of air; for a current of air impinging against rising ground must, in order to pass over it, necessarily rise. So far as the author knows, however, no attempt has yet been made to investigate this process quantitatively, except perhaps, for the stratum of air immediately contiguous to the earth, whose ascension being equal to that of the surface itself, is thereby known directly. Such a quantitative treatment will be attempted in the following article. Even although this is only possible under special assumptions which represent nature with the closest approximation, it will, however, always offer a practical basis for estimating the purely mechanical influence exerted by the configuration of the surface of the earth on the formation of rain.

1.

In order to find the standard vertical components of the velocity of the air currents that determine the condensation, we must, first of all, solve the hydrodynamic problem of the movement of the air over a rigid surface of a given shape. In this connection we must make a series of simplified assumptions, as follows:

1. The current must be stationary; 2, it must be continuous and free from whirls; 3, it must flow everywhere parallel to a definite vertical plane, and consequently depend only on the vertical coordinate (y), and one horizontal coordinate (x); 4, the internal friction, as well as the external (or that due to the earth's surface), may be neglected; 5, at great heights there must prevail a purely horizontal current of constant velocity, a. As to the configuration of the ground, we must, corresponding to proposition 3, assume that the profile curves are identical in all vertical planes that are parallel to the plane of $x\,y$; 6, and further, we assume the surface profile to be *periodic*, that is to say, the surface of the earth is formed of similar parallel waves of mountains without, however, determining in advance the special equation of the profile curves.

If we designate by u and v the horizontal and vertical components of velocity and by ε the density, then, in consequence of assumptions 1 and 3, there follows the condition

$$\frac{\partial(\varepsilon u)}{\partial x}+\frac{\partial(\varepsilon v)}{\partial y}=0$$

and in consequence of 2 there must exist a velocity potential, φ, which, according to 3, can only depend upon x and y, so that

$$u=\frac{\partial\varphi}{\partial x}\quad v=\frac{\partial\varphi}{\partial y},\text{ and }\frac{\partial}{\partial x}\left(\varepsilon\frac{\partial\varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varepsilon\frac{\partial\varphi}{\partial y}\right)=0.$$

If we consider that the density of the air in a horizontal direction (excluding large differences of temperature at the same level) changes much more slowly in a horizontal than in a vertical direction, then we can regard ε as a function of y only, and obtain for φ the differential equation—

$$(1)\qquad \varepsilon\,\Delta\varphi=-\frac{\partial\varepsilon}{\partial y}\frac{\partial\varphi}{\partial y}.$$

The law of the diminution of density with altitude will, strictly speaking, be different for each particular case, because the vertical diminution of temperature in a rising current of air, which determines the rate of diminution of density, depends upon the condensation. But it is allowable, as a close approximation and as is usually done in barometric hypsometry, to assume the law of diminution of pressure which obtains, strictly speaking, for a constant temperature only, and which, as is well known, reads as follows:

$$\text{nat log}\frac{p_0}{p}=q\,y,$$

where q is a constant and has very nearly the value of 1/8000 if y, the difference in altitude, be expressed in meters. In this case the following also holds good:

$$\log\frac{\varepsilon_0}{\varepsilon}=q\,y,$$

and, consequently,

$$-\frac{1}{\varepsilon}\frac{\partial\varepsilon}{\partial y}=q;$$

hence the differential equation for φ becomes

$$(2)\qquad \Delta\varphi=q.\frac{\partial\varphi}{\partial y}.$$

A solution of this differential equation that satisfies the assumptions 5 and 6, is given by the expression

(3) $$\varphi = a\,(x - b\cos m\,x.\,e^{-ny}),$$

in which the following relation exists between the constants m and n.

(4) $$\begin{cases} m^2 - n^2 = q\,n\,; \\ n = -\dfrac{q}{2} + r, \text{ where } r = \sqrt{m^2 + q^2/4}. \end{cases}$$

In order to ascertain what profile or configuration of the ground corresponds to the current determined by this velocity potential, we must look for the lines of flow; for one of these must certainly agree with the profile curve. The differential equation of the stream lines reads as follows:

$$dy : dx = \frac{\partial \varphi}{\partial y} : \frac{\partial \varphi}{\partial x} = a\,b\,n \cos m\,x\,.\,e^{-ny} : a\,(1 + b\,m \sin m\,x\,.\,e^{-ny}).$$

The integration of this equation gives

(5) $$e^{-ny}.\sin m x = -\frac{m}{b\,q\,n} + B\,e^{qy},$$

wherein B represents the parameter of the stream lines.

If we assume that the curve of the profile of the surface passes through the points $x = 0$ and $y = 0$, then for these values $B = m / b\,q\,n$, and if its ordinates are designated by η, its equation becomes

$$b\,\frac{q\,n}{m} \sin m\,x.\,e^{-n\eta} = e^{q\eta} - 1$$

or

$$b\,\frac{n}{m} \sin m\,x.\,e^{-r\eta} = \frac{e^{\frac{q}{2}\eta} - e^{-\frac{q}{2}\eta}}{q}.$$

As long as η remains so small that for both the highest and lowest points of the profile of the surface of the earth $(q\eta/2)^2$ is negligible in comparison with unity — which is practically always the case for the mountains that come under our consideration — we can write

(5′) $$\eta = b\,\frac{n}{m} \sin m\,x.\,e^{-r\eta}; \quad \begin{bmatrix} n = -\dfrac{q}{2} + r, \\ r = \sqrt{m^2 + q^2/4} \end{bmatrix}.$$

In these expressions b and m appear as parameters that can be chosen at will, the first of which determines the altitudes and the second the horizontal distances between the mountain ridges; we have, namely, $m = 2\,\pi/\lambda$, if λ denotes the wave length, that is to say the distance between two corresponding points, as for example the summits of neighboring mountain ranges.

It is easy to show that the stream line determined by the velocity potential (3) for the configuration of the ground given by the transcendental equation (5′) is the only one compatible with the general conditions 1 to 5. Moreover, since a potential current is determined single valued, for the interior, by the value of $\frac{\partial \varphi}{\partial n}$ along the boundary of a closed region, therefore, our solution in case it gives horizontal velocities that are constant, or slowly diminish with the altitude above the center of the valley, is also applicable to the specially interesting practical case in which only one single mountain range rises above an extended plain and is struck perpendicularly by a uniform horizontal current of air. To what extent this holds good must be established in each special case.

The horizontal and the vertically upward velocity components corresponding to our solution are:

(6) $$u = a\,(1 + b\,m \sin m\,x\,.\,e^{-ny})$$

(7) $$v = a\,b\,n \cos m\,x\,.\,e^{-ny}.$$

It would now be desirable, in order to be able to handle the cases actually occurring in nature, to adapt our solution to some form of the earth's surface arbitrarily chosen. The first thought would be to attempt this by the superposition of a series of velocity potentials of the form of equation (3) having different constants m and b, or in other words to write

(8) $$\varphi = \sum \varphi_h = a\left\{x - \sum^h b_h \cos m_h x\,.\,e^{-n_h y}\right\};$$

but we find that this solution only corresponds to a superposition of the profile curves, that is to say, it gives

(9) $$\eta = \sum \eta_h = \sum b_h \frac{n_h}{m_h} \sin m_h\,x\,.\,e^{-r_h \eta}$$

only when we can put the exponential functions $e^{-n_h y}$ and $e^{-r_h \eta}$ both equal to unity. In this case η is at once transformed into the simple trigonometrical series

(9′) $$\eta = \sum^h b_h \frac{n_h}{m_h} \sin m_h x$$

and therefore, by putting $m_h = h\,m_1$ we can develop any arbitrary function, $\eta = f(x)$, into a series, proceeding for any value of x greater than zero and less than $\lambda/2$. But the condition that $e^{\pm h m \eta}$ is equal to unity for any large value of the quantity h will not be fulfilled for any arbitrary form of the profile curve if its maximum altitude is assumed to be very small in comparison with the wave length λ. Therefore, we must limit ourselves to an approximate representation of the desired profile curve by a definite number of terms of the series that enters equations (9) or (9′). Especially can we in this way never attain the rigid solution for a ground profile that has sharp angles. However, the neglected higher terms of the series have a proportionately slighter influence on the vertical velocity at great altitudes and, therefore, on the resulting precipitation, in proportion as their serial number h is larger.

2.

As a first example, we choose a form of profile to correspond as closely as possible to a plane, broad valley and a plateau like mountain range, because, in this case, we may expect nearly the same conditions on the slope of the mountain as if it were struck by a uniform horizontal current of air. A profile curve of this kind, which rises steadily between the values x greater than $-\frac{\lambda}{12}$ and less than $+\frac{\lambda}{12}$ and falls also with uniform gradient between the limits $x = 5/12\,\lambda$ and $x = 7/12\,\lambda$, and in the intermediate region describes a horizontal straight line at the distance $+H$ from the axis of x, is obtained by means of the Fourier series

$$\eta = \frac{24\,H}{\pi^2} \sum^h \frac{1}{h^2} \sin \frac{h\,\pi}{6} \sin \frac{2\,h\,\pi}{\lambda}\,x,$$

where h has all positive uneven numbers. In order to represent a profile curve of the given form approximately, we take the first three terms of the series, and therefore have

(10) $$\eta = C\left\{\tfrac{1}{2} \sin m_1 x + \tfrac{1}{9} \sin 3\,m_1 x + \tfrac{1}{50} \sin 5\,m_1 x\right\}.$$

The numerical values of the parameters are:

$$\lambda = 60{,}000 \text{ meters, also } m_1 = \frac{2\,\pi}{\lambda} = 0.1047 \times 10^{-3}$$

and

$$C = 1{,}100 \text{ meters.}$$

The coefficients b_h, in the expressions (8) and (9) therefore, have the following values:

$$b_1 = 881,\ b_3 = 148.3,\ b_5 = 24.8$$

The profile given by equation (10) is shown in fig. 1, where the vertical scale is magnified five times. We perceive that the ascending gradient is nearly all confined to the interval between

$$x \text{ greater than } -\frac{\lambda}{12} \text{ and less than } +\frac{\lambda}{12}$$

where, moreover, it is quite uniform, and further, that the surface of the valley is raised a little in the center, and the surface of the plateau mountain is depressed by the same amount. The difference in altitude between the center of the valley and the center of the mountain, which according to the adopted numerical values should be 900 meters, is therefore, not the absolute maximum difference but is about 18 meters less. The profile curve here considered corresponds indeed, according to what has been above said, only approximately to the velocity potential

$$(11) \quad \begin{cases} \varphi = a \left\{ x - b_1 \cos m_1 x \, . \, e^{-n_1 y} - b_3 \cos 3m_1 x \, . \, e^{-n_3 y} \right. \\ \qquad \left. - b_5 \cos 5m_1 x \, . \, e^{-n_5 y} \right\}, \end{cases}$$

as determined by the above coefficients, b_h, but we can easily demonstrate that in the present example the differences could scarcely be observed in fig. 1.

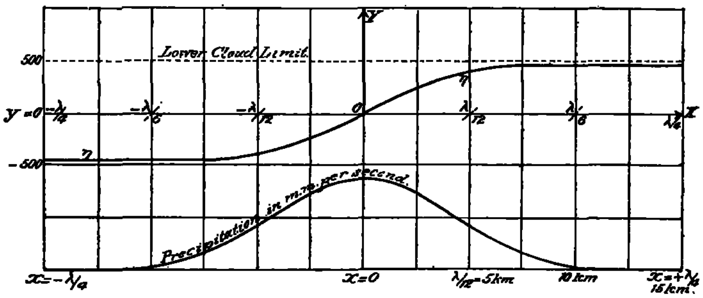

FIG. 1.

From the preceding value of φ we derive the following values for the components of the velocities of the current:

$$(12) \quad \begin{cases} u = a \left\{ 1 + \sum b_h \, m_h e^{-n_h y} \sin m_h x \right\} \\ \quad = a \left\{ 1 + \frac{2\pi}{\lambda} \left(b_1 e^{-n_1 y} \sin m_1 x + 3 \, b_3 \, e^{-n_3 y} \sin 3 \, m_1 x \right. \right. \\ \qquad \left. \left. + 5 \, b_5 \, e^{-n_5 y} \sin 5 \, m_1 x \right) \right\}, \end{cases}$$

$$(13) \quad \begin{cases} v = a \times \sum b_h \, n_h \, e^{-n_h y} \cos m_h x \\ \quad = a \times 0.1152 \left\{ \frac{1}{2} e^{-n_1 y} \cos m_1 x + \frac{1}{3} e^{-n_3 y} \cos 3 \, m_1 x \right. \\ \qquad \left. + \frac{1}{10} e^{-n_5 y} \cos 5 \, m_1 x \right\}. \end{cases}$$

These equations show that when $x = 0$, that is to say above the center of the slope of the mountain, u is a constant $= a$ at all altitudes; above the valley where x is less than 0, u is smaller than a; and above the mountain, or plateau, where x is greater than 0, u is larger than a; the constant a can also be considered the mean horizontal velocity at any given altitude.

For different altitudes H above the center of the valley we have the following values:

$H = 450 + y$:	0	450	2,000	5,000
$\frac{u-a}{a}$:	-0.068	-0.0676	-0.0675	-0.0646

Therefore, up to the altitude of 5,000 meters, the horizontal velocity is sensibly constant and the vertical velocity 0; and, according to what is said in reference to equation (5′) our solution holds good for the case when the profile is continued as a horizontal straight line indefinitely toward the negative side from the point $x = -\lambda/4$, and above this there flows a truly horizontal current of air whose velocity is sensibly constant, namely, 0.93 a up to an altitude of 5,000 meters and increases in the strata above that until it attains the value a.

Above the mountain, as at the point where $x = +\lambda/4$, the velocities, u, are greater than a by nearly as much as they are smaller above the valley.

The distribution of the vertical velocity component which determines the condensation of aqueous vapor is a more complicated matter. In order to represent it, let the values of v/a for different values of the coordinates x and y be as given in the following table:

y	x				
	0	$\pm\frac{\lambda}{12}$	$\pm\frac{\lambda}{8}$	$\pm\frac{\lambda}{6}$	$\pm\frac{\lambda}{4}$
500	0.099	0.0406	0.0129	-0.0012	0
1,530	0.0842	0.04075	0.0149	$+0.00226$	0
2,440	0.0740	0.0400	0.0182	0.0064	0
3,460	0.0651	0.0387	0.0206	0.0093	0
4,530	0.0575	0.0370	0.0217	0.0108	0

Therefore, whereas there is a steady decrease of v with altitude above the center of the slope of the mountain, on the other hand these vertical velocities increase with the altitude in the neighborhood of the foot of the mountain as well as on the plateau at the point $x = \pm \lambda/8$ up to a maximum at some very great altitude. (The isolated negative value that occurs for $x = \lambda/6$ and $y = 500$ is explained by the above-mentioned slight depression of the summit of the plateau mountain.)

In order, now, to investigate the condensation of aqueous vapor that occurs in consequence of the ascending currents of air forced upward by the upward slope of the ground, we first make the assumption that the ascending mass of air experiences an adiabatic change of condition and that adiabatic equilibrium prevailed already in the horizontal current of air advancing toward the slope of the mountain. In this case the air will be everywhere saturated at a certain altitude that can be computed from the temperature and humidity of the air at the surface of the valley. In a unit of time the quantity of air, $v \varepsilon$, flows in a vertical direction through a space having a unit of horizontal surface and an altitude $d y$. If this element of space lies above the lower limit of the clouds, then in this quantity of air there will be as much aqueous vapor condensed as the difference between what it can contain in the state of saturation at the altitude $y + d y$ and what it can contain at the altitude y. Therefore this quantity is

$$v \, \varepsilon \, . \, \frac{-\partial F}{\partial y} d y,$$

where $F(y)$ is the specific humidity of saturated air at the altitude y.

Still assuming a stationary condition, we have—

$$(14) \qquad W = -\int_{y_0}^{y'} v \, \varepsilon \, F'(y) \, d y,$$

as representing the total quantity of aqueous vapor condensed in a unit of time in a stratum of cloud above the unit of basal area between the altitudes y_0 and y'.

This would also be equal to the quantity of precipitation falling from that layer of cloud on to the unit of horizontal base in case the products of condensation simply fell vertically without being carried along by the horizontal current of air. We will make this assumption, since as yet we have no clue by which to frame a computation of the horizontal transportation of the falling particles of precipitation. It is, however, easy to foresee that the horizontal transportation would be of importance, especially for the slowly-falling particles of water or ice in the upper strata of clouds, and that on the other hand, the larger drops that carry down with themselves the water condensed in the lower strata of clouds will fall at a relatively slight horizontal distance. But now, as the numerical computation shows, the lower cloud strata contribute relatively far more to the condensation than the upper clouds; therefore, the influence of the horizontal transport will not be so very large, at least with moderate winds. Moreover, this influence does not affect the total quantity of precipitation caused by the flow up the mountain side, but only its distribution on the mountain slope and it consists essentially in a transfer of the location of maximum precipitation toward the mountain. In this sense, therefore, we have to expect a departure of the actual distribution of precipitation from that which is theoretically given by the computation of W as a function of x, according to equation (14). This departure will, under otherwise similar circumstances, be considerably larger in the case of snowfall than in the case of rain.

As concerns the upper limit y', which is to be assumed in the integration of equation (14) in order to obtain the total quantity of precipitation falling upon a unit of surface, we have to substitute for y' that altitude at which condensation actually ceases in the ascending current of air. Theoretically, if from the beginning adiabatic equilibrium prevails up to any given altitude, then the condensation brought about by the rising of the earth's surface must also extend indefinitely high, even to the limit of the atmosphere, since the vertical component of velocity diminishes asymptotically toward zero. But practically, our solution of the problem of flow no longer holds good for very high strata probably, and certainly the assumption of adiabatic equilibrium does not hold good, and even if the latter were the case, if therefore, the ascending current carried masses of air from the surface of the earth up to any given altitude, still, in consequence of the increasing weight of the particles of precipitation carried up by the ascending current on the one hand, and the increasing insolation on the other hand, an upper limit of cloud must be formed.[1]

We will therefore assume as given some such upper limit of clouds at a definite altitude, and for simplicity will assume this to be the same everywhere. The value of this altitude, y', is the upper limit of the integral (14). However, the altitude assumed for y' if it is large, namely, many thousands of meters, can have only a slight influence on the value of W, since both $-F'(y)$ and $v\varepsilon$ rapidly diminish with the altitude.

For the numerical computation of W, it is advantageous to first bring the expression (14) by partial integration into the following form:

$$(14a)\qquad W(x)=\Big[v\varepsilon F(y)\Big]_{y'}^{y_0}+\int_{y_0}^{y'}F(y)\frac{\partial\varepsilon v}{\partial y}dy.$$

In this expression v is given by equation (13) as a function of y and x. $F(y)$, or the saturation value of the specific moisture at the altitude y, as well as the corresponding values of the pressure and temperature necessary for the computation of ε are most easily obtained with the help of the graphic diagram for the adiabatic changes of condition of moist air first given by H. Hertz, since a simple analytical expression for these quantities can not be given. In using the Hertzian table[2] we have to remember that y is not the absolute altitude but the altitude above the axis of x in our system of coordinates, therefore, in order to obtain the altitude above sea level, it is still to be increased by the quantity $-\eta\,(_{x=-\frac{\lambda}{4}})$ and also by the altitude of the valley above the sea. The integral in equation (14a) can be evaluated with sufficient accuracy by dividing the integral from y_0 to y' into parts $y_0\ldots y_1, y_1\ldots y_2$ $y_{h-1}\ldots y_h\ldots$ (where $y_h=y'$), and for each of these introducing an average value F_{mk} whereby we obtain equation (15).

$$(15)\qquad \int_{y_0}^{y_h}F(y)\frac{\partial(\varepsilon v)}{\partial y}dy=\sum_0^h{}_k F_{m_k}\Big[(\varepsilon v)_k-(\varepsilon v)_{k-1}\Big].$$

In order to execute the complete computation of W for a special example, we will assume that the current of air which strikes the mountain having the profile shown in fig. 1 has a pressure of 760 millimeters, temperature, 20°, and specific humidity, 9.0,[3] at the bottom of the valley. Hence, according to our assumption of adiabatic equilibrium it follows that the lower limit of the clouds will lie at an altitude of 950 meters above the bottom of the valley, and, therefore, 50 meters above the center of the mountain, if $y_0=500$; the specific humidity is at this cloud level, $F(y)'=9.0$, and the temperature is 11° C. We will further assume that the upper limit of the clouds is at an altitude of about 5,000 meters, or $y'=4{,}530$ meters, where the temperature has sunk to $-13.6°$ and the specific humidity to $F(y)=2.5$. At the altitude of 3,000 meters the temperature 0° C. is attained. The application of the Hertzian tables assumes that for temperature below 0° C. the product of condensation is ice; whether this is really true is at least questionable for moderately low temperatures, but the assumption that water below the freezing point is precipitated will not change the results very much. Since corresponding to the assumed stationary condition, we have to assume that all condensed water immediately falls from the clouds; therefore, in our computation we have to omit the hail stage of Hertz, in which the water that is carried along with the cloud is frozen[4].

For the computation of the integral according to equation (15) the cloud is divided into four layers whose mutual boundaries or limits occur at $y_1=1{,}530$, again $y_2=2{,}440$, and $y_3=3{,}460$ meters; for these altitudes we have $\varepsilon=1.00$ and 0.912 and 0.816, and corresponding to these $F(y)=6.9$ and 5.35 and 3.8.

We thus find the following values for W/a:

$x=$	0	$\pm\frac{\lambda}{12}$	$\pm\frac{\lambda}{8}$	$\pm\frac{\lambda}{6}$
$\frac{W}{a}=$	0.475	0.241	0.0985	0.0081 grams per second

per square meter.

From this table we obtain the depth of the precipitation in millimeters per hour by multiplying by 3.6; the result is shown in the lower curve of fig. 1. The values of the precipitation for a mean horizontal velocity of the current of 1 meter per second are as follows:

$x=$	0	$\pm\frac{\lambda}{24}$	$\pm\frac{\lambda}{12}$	$\pm\frac{\lambda}{8}$	$\pm\frac{\lambda}{6}$	$\pm\frac{\lambda}{4}$
$W'=$	1.71	1.47	0.867	0.355	0.029	0

[1] W. von Bezold. Sitzb. Ber. Akad. Wiss., Berlin, 1888, p. 518, and 1891, p. 303.

[2] H. Hertz. Met. Zeit., 1884. Vol., I pp. 421–431.

[3] That is, 9.0 grams of water per kilogram of air.

[4] The influence upon the adiabatics of condensation, whether we assume, as in the Hertzian table, all condensed water to be carried with it or to immediately fall away, is of no importance in the present problem.

Hence, the precipitation is heaviest above the middle of this slope of the mountain, where for the very moderate wind velocity of 7 meters per second, it attains the very considerable rate of 12 millimeters per hour. In this connection it is, indeed, to be remembered that we have assumed exceptionally favorable conditions for the precipitation in that we have assumed the onflowing air to have been already fully saturated throughout the whole 4,000 meters in depth of the layer between y_0 and y'.

The comparison of the curve of precipitation with the curve of profile in fig. 1 shows that although the maximum of precipitation coincides with the maximum gradient of the slope of the mountain, yet the depth of precipitation diminishes more slowly toward the plane of the valley and the plateau of the mountain than does the slope of the earth's surface; thus, for instance, the latter slope at the point where $x = \pm \lambda/12$, and which is given by $\partial \eta / \partial x$, amounts only to 1/20 of the maximum slope, while the precipitation at this point is more than 1/5 of its maximum value. Therefore, under the conditions here assumed, the effect of a mountain slope in producing precipitation makes itself felt in the plain lying in front of the foot of the slope. All of which agrees with actual experience[5]. The fact that in reality the maximum precipitation appears to be pushed more toward the ridge of the mountain is certainly partly explained, as well as suggested, by the horizontal transportation of the products of condensation in the clouds, but also in part by the departure of the real distribution of temperature and moisture from that here assumed. (See Section 4 hereafter.)

The determination of the total quantity of precipitation caused by the mountain slope will be attained if we integrate the value of W as determined by equation (14) as a function of x between the limits $x = -\lambda/4$ and $x = +\lambda/4$. The result is, therefore,

$$(16) \qquad G = \int_{-\frac{\lambda}{4}}^{+\frac{\lambda}{4}} W(x)\, d\, x = -\int_{y_0}^{y'} \varepsilon F'(y) \int_{-\frac{\lambda}{4}}^{+\frac{\lambda}{4}} v\, d\, x.$$

In this equation, according to equation (13) we have:

$$\int_{-\frac{\lambda}{4}}^{+\frac{\lambda}{4}} v\, d\, x = a \times 1{,}100 \left\{ e^{-n_1 y} - \frac{2}{9} e^{-n_3 y} + \frac{1}{25} e^{-n_5 y} \right\}.$$

For our present example we find $G = 5{,}100a$ grams per second over a strip 1 meter wide and about 22 kilometers long. Hence, there follows for the average precipitation for the whole mountain slope

$$W'_m = 0.833a \text{ millimeters per hour.}$$

3.

In the example we have just discussed the lower limit of the clouds was higher than the summit of the mountain. If the reverse is the case, then, for that portion of the mountain slope that is immersed in the clouds we must take η instead of y_0 as the lower limit of the integral in the formulæ (14) to (16); therefore, the theoretical distribution of precipitation would no longer be symmetrical with respect to the zero point on the axis of abscissas. As an example of this case we will consider the flow of air above the ground profile that is represented by the simple equation

$$\eta = C \sin m x \,.\, e^{-r\eta}.$$

As to the constants we will adopt the following:

$C = 1{,}000$ meters, $\lambda = 24{,}000$ meters;
hence $m = 0.262 \times 10^{-3}$, $r = 0.269 \times 10^{-3}$,

[5] Hann. Climatology, 2d edition, vol. 1, p. 295; also Assmann, Einfluss der Gebirge auf das Klimat von Mittel Deutschland, 1886, p. 373.

and for the vertical coordinate η we find from equation (5)

for $x =$	$-\frac{\lambda}{4}$	$-\frac{\lambda}{6}$	$-\frac{\lambda}{12}$	0	$+\frac{\lambda}{12}$	$+\frac{\lambda}{6}$	$+\frac{\lambda}{4}$
$\eta =$	$-1{,}495$	$-1{,}194$	-585	0	$+444$	$+715$	$+805$ meters.

The resulting curve is shown in fig. 2. The altitude of the summit of the mountain above the plain of the valley amounts to 2,300 meters. The valley may be 100 meters above sea level; the atmospheric pressure in the valley is assumed at 750 millimeters, the temperature 23°, and the specific humidity 10 grams of water per kilogram of air. From the Hertzian table we find the lower cloud limit at the altitude of 1,220 meters, that is to say at $y = -375$. The upper limit of the clouds is assumed at $y' = 2{,}400$ and, therefore, at 4,000 meters above sea level. Therefore, for that portion of the clouds lying below the summit of the mountain, which is limited to the negative values of the abscissas up to $x = -1.35$ killimeters approximately, since according to equation (7)

$$v = C a m \cos m x \,.\, e^{-ny}$$

we have:

$$W = -\int_{y_0}^{y'} \varepsilon\, v\, F'\, d\, y = -a\, C\, m \cos m\, x \int_{y_0}^{y'} \varepsilon\, F'(y)\, e^{-ny} d\, y$$
$$= a \cos m x \times 1.09.$$

Therefore, the depth of the precipitation will here be represented by a simple cosine curve and, in general, corresponds to the slope of the mountain, which is computed from equation (5′) by the expression:

$$\frac{d\eta}{dx} = \frac{C\, m \cos m\, x \,.\, e^{-r\eta}}{1 + C\, r \sin m\, x \,.\, e^{-r\eta}}.$$

For the region lying above the lower cloud limit y_0 the value of $W(x)$ can not be represented by a simple function of x. We find the precipitation in millimeters per second for a horizontal velocity $a = 1$, as follows:

For $x =$	-6	-5	-4	-3	-2	Lower half of the cloud.
$W =$	0	1.01	1.96	2.78	3.40	
For $x =$	-1	0	$+2$	$+4$	$+6$	In the cloud.
$W =$	3.50	2.94	1.95	0.88	0	

The distribution of precipitation, as given by these figures is shown in fig. 2 by the curve of dashes. The curve of dots represents the symmetrical line that would obtain if the mountain were not immersed in the clouds. The location of maximum precipitation is 3.93 for $x = 0$ and is 3.68 for $x = -6.3$.

The total quantity of precipitation is computed by the formula:

$$G = -a\, C \sin m x \int_{y_0}^{y'} \varepsilon\, F'(y)\, e^{-ny} d\, y$$

and is approximately equal to 22,730; this is distributed over a horizontal strip 12,000 meters in length, and therefore, for a uniform distribution for $a = 1$ the precipitation averages 1.9 millimeters.

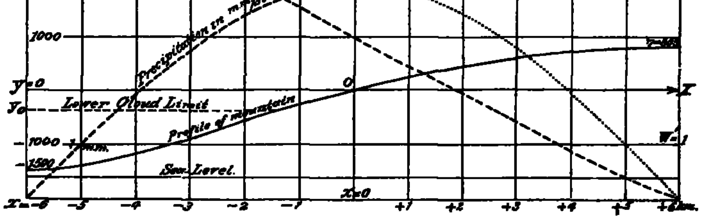

FIG. 2.

From the preceding expression for G, it is plain that for any given altitude of the mountain summit G will be smaller the shorter and steeper the slope becomes, that is to say, the smaller the value of λ is, since the exponent ny increases with diminishing values of λ. In the present case the horizontal velocity of the wind is given by the expression:

$$u = \frac{\partial \varphi}{\partial x} = a\left(1 + C\frac{m^2}{n}\sin\ m\ x.\ e^{-ny}\right)$$

$$= a\,(1 + 0.332\ \sin\ m\ x.\ e^{-ny});$$

which attains its minimum, $= 0.547\,a$, at the bottom of the valley, and its maximum, $1.283\,a$, at the summit of the mountain, and has a for the mean value of all the horizontal planes. Above the center of the valley it increases gradually with altitude, asymptotically approaching its limiting value, a; for example, at the level $y = 0$, it is equal to $0.668\,a$, and at the level $y = 2{,}400$ it is already equal to $0.80\,a$. Therefore, if the stream under consideration proceeds from a point $x = -\lambda/4$, as a purely horizontal current of air flowing over a plain, then its velocity must diminish with the altitude in the ratio e^{-ny}. This would, of itself, be a plausible assumption, but there would then be a vortex motion for each horizontal current of air, which can not, strictly speaking, continue steadily in the above assumed potential motion.

4.

The assumptions hitherto made by us, namely, that the distribution of temperature in the current of air that impinges upon the mountain side already corresponds to the condition of indifferent equilibrium, that is to say that it is the same as would occur in an ascending current of air under adiabatic changes of condition, is in general not actually fulfilled. The scientific balloon ascensions at Berlin have recently given us reliable conclusions as to the real conditions of temperature and moisture in the free atmosphere up to altitudes of 8,000 meters. The mean values of the temperature and moisture at successive levels, 500 meters apart, which von Bezold has deduced[6] from the observations of Berson and Süring show that the mean vertical diminution of temperature is slower than the adiabatic, and that, in general, the moisture does not attain the saturation value. In a horizontal current of air, in which these average conditions prevail, the air will, therefore, never be saturated, and, consequently, our assumption of the existence of a constant lower limit to the clouds is not allowable. Moreover, it is no longer the vertical component alone that controls the condensation that shall occur at any given point in the current of air ascending above the mountain slope, as was assumed in the derivation of formula (14). We must rather, in the computation of W, consider that the quantity of water condensed in a unit of space under steady stationary conditions is equal to the excess of the quantity of water vapor flowing into the space above that simultaneously flowing out. For one cubic meter and one second this excess is:

$$-\left(\frac{\partial(\varepsilon\,u\,F)}{\partial x} + \frac{\partial(\varepsilon\,v\,F)}{\partial y}\right),$$

or since because of the equation of continuity we have approximately

$$\frac{\partial\,\varepsilon\,u}{\partial x} + \frac{\partial\,\varepsilon\,v}{\partial y} = 0,$$

therefore[7],

$$-\,\varepsilon\left(u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y}\right),$$

and hence,

$$(17)\qquad W = -\int_{y^0}^{y'} \varepsilon\left(u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y}\right)d\,y,$$

where y^0 and y' indicate the altitudes of the limits of the clouds above the point under consideration. The evaluation of the integral still demands not only a complete knowledge of the stream, but also the determination of the cloudy region, that is to say, that region in which the atmosphere is saturated and the distribution of temperature therein, since the latter first gives us the value of F. To this end we have to follow the adiabatic change of condition of the air around each curve of flow, starting with the given temperature and humidity in the vertical above the center of the valley where $x = -\lambda/4$, where the current is truly horizontal.

By connecting together those points in the individual stream lines at which saturation is just attained we find, first, the contour of the cloudy region.

Since the form of the clouds is also of interest in and of itself[8], therefore its determination will be carried through as a part of our second example, in that above the center of the valley, where $x = -\lambda/4$ first for the summer, then for the winter, we make some assumption as to the mean distribution of temperature in accordance with von Bezold's collected data, on page 21 of his memoir above quoted. In accordance with this, we have:

	For $y =$	−1,500	−600	+400	+1,400	+2,400 m.
		Valley above sea level, 100 m.			Height above sea level, 4,000 m.	
Summer	$t =$	17.7°	11.0°	5.3°	+ 0.9°	− 5.0°
	$F =$	8.2	6.69	4.59	3.03	2.60*
Winter	$t =$	0.2°	−0.6°	−5.1°	−10.8°	−14.6°
	$F =$	2.92	2.17	1.64	1.19	0.86

In place of the value of F, designated by a star, we will take that value (2.2) that results from the smoothing out of the protuberant corners which the curve for F (see von Bezold, fig. 11) shows at the altitude of 4,000 meters.

According to equation 5 the lines of flow have for their expression

$$e^{-ny}\sin m\,x = -\frac{m}{b\,q\,n} + B\,e^{qy},$$

or if y_0 is the value of y when $x = 0$, and $y - y_0 = \eta$, there results,

$$e^{-n\eta}\,e^{-ny_0}\sin m\,x = \frac{m}{b\,q\,n}\,(e^{q\eta} - 1),$$

$$\frac{b\,n}{m}\,e^{-ny_0}\,e^{-r\eta}\sin m\,x = \frac{1}{q}\left(e^{\frac{q\eta}{2}} - e^{-\frac{q\eta}{2}}\right).$$

With the same degree of approximation as before the right-hand side of this equation can be put equal to η; therefore the equation takes the following form:

$$(18)\qquad \eta = b\frac{n}{m}\sin m\,x\,.\,e^{-r\eta}\,e^{-ny_0},$$

which differs from equation (5′) of the profile curve of the ground only through the factor which is constant for each line of flow, which factor causes the amplitude of the waves to steadily diminish upward.

If, now, the lines of flow are made through a definite point y'_h for the vertical and $x = -\lambda/4$, then for this point we de-

[6] W. von Bezold. Theoretische Betrachtungen, etc. Theoretical considerations relative to the results of the scientific balloon ascensions of the German Association for the Promotion of Aernautics at Berlin. Brunswick, 1900, pp. 18–21.

[7] In so far, namely, as the quantity of the aqueous vapor condensed in a unit of volume is inappreciably small in comparison with the total quantity of moist air flowing through this space.

[8] It seems, for example, quite possible to argue from the observed boundary of the clouds inversely to the percentage of moisture in the current of air flowing toward the mountain slope.

termine the appropriate value η' from the transcendental equation:

$$(19) \qquad \eta' = -b \frac{n}{m} e^{-r\eta'} e^{-n(y'_h - \eta')}$$

and then substitute $y^0_h = y'_h - \eta'$ in equation 18.

In this way we have computed the four lines of flow whose initial and lowest points are at the altitude above sea level of 1,000, 2,000, 3,000, and 4,000 meters, and which are drawn as curves I, II, III, IV, in fig. 3. The highest points of these curves are at the altitudes 2,940, 3,610, 4,333, 5,100 meters, respectively.

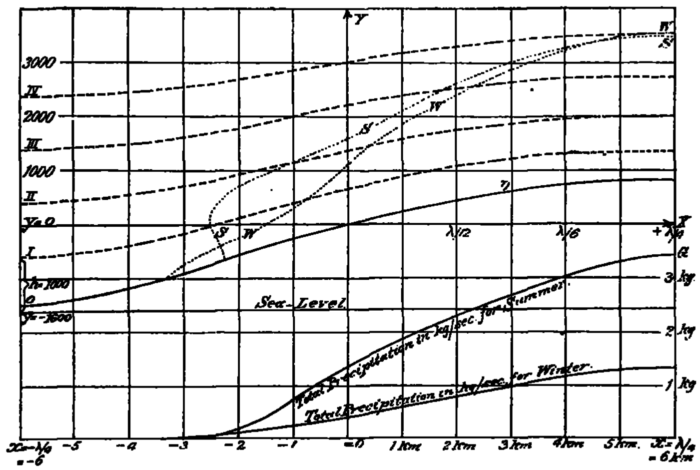

FIG. 3.

If now, by means of the Hertzian table, we determine the altitudes at which condensation begins at the base curve 0 and for the curves I, II, III, IV, then assuming the above given values of t and F,[9] we find the following results:

	0	I	II	III	IV
For the summer	930	1,570	2,730	4,060	(5,125)
For the winter	600	2,070	3,100	4,130	5,100

In the summer, according to this table, condensation will not take place on the stream line IV, since its summit lies at the altitude of 5,100 meters; the summit of the clouds will, therefore, lie a little below this. In the winter, the summit of line IV accidentally agrees with the summit of the cloud. In the construction of the cloud limit, introduced as a dotted line in fig. 3, and indicated by S for summer and W for winter, we have also used the lines of flow midway between 0 and I and I and II, respectively.[10]

We can now, with the help of the Hertzian table, easily find the quantity of water condensed in every kilogram of moist air as it progresses along any one of the lines of flow that we have constructed, either in its totality or as it passes successive vertical lines: we thus attain the following values of the total condensation:

Curve	0	I	II	III
For the summer	2.85	2.42	1.22	0.26 grams.
For the winter	1.5	0.74	0.34	0.14 grams.

[9] From the above numbers it follows that an elevation of any form of less than 500 meters will not give occasion for condensation under average atmospheric conditions neither in summer nor in winter. In the summer, for a mountain altitude of between 600 and 800 meters, a cloud will form between the altitudes 1,000 and 3,000 meters, but will not touch the mountain; it is only for greater mountain heights that the cloud will rest on the mountain.

[10] In an analogous way for the first example, where we have assumed a plateau-like mountain of 900 meters altitude, we find a region of cloud which, for the average summer conditions, begins at 40 meters below the summit of the plateau and reaches up to over 3,000 meters; but in winter, on the other hand, it begins at 500 meters above the valley and rises up only about 700 meters above the mountain top; therefore, in this season it covers the mountain like a flat cap.

Let $g_x(h)$ be the quantity condensed up to the abscissa x when moving along that line of flow whose initial point is at the altitude h, and let H be the initial altitude of that line of flow which at the given abscissa x intersects the upper cloud limit; moreover, let u' be the horizontal velocity of flow and ε' the density of the air at the altitude h above the bottom of the valley, therefore, for the point whose abscissa $= -\lambda/4$; then will the total quantity condensed per second above the base area one meter broad from the beginning of the clouds to the point x, expressed in grams, be as follows:

$$(20) \qquad G_x = \int_0^H \varepsilon' u' g_x(h)\, dh.$$

The quantity of air, εu kilograms, flows in one second through a strip of the vertical plane at $x = -\lambda/4$, having a unit width and the height dh; but an equal quantity must flow out per second through the vertical whose abscissa is x, and since the condition is steady, it therefore behaves as though the quantity of air, εu, had moved in one second along the line of flow from $-\lambda/4$ up to x; but in this the quantity of water $\varepsilon u g_x(h)$ is separated from the air according to our definition of g.

If we have computed G as a function of x, according to to formula (20), then, finally, we have

$$(21) \qquad W = \frac{\partial G}{\partial x}$$

as the quantity of water, expressed in grams, per horizontal square meter per second, that falls at the place x. In this way the determination of W is executed more conveniently than through the direct formula (17). By assuming the average conditions for the summer in the above example for $a = 1$, we find that the integral (20), if we compute it as approximately equal to the sum of the intervals between the individual current curves of flow as constructed, gives the following:

$$G_{x=0} = 1{,}352, \quad G_{x=\frac{\lambda}{8}} = 2{,}680, \quad G_{x=\frac{\lambda}{4}} = 3{,}460 \text{ grams.}$$

This last number indicates the total precipitation falling on a strip one meter wide in one second on the side of the slope that faces the wind. According to the course of the curve SS, as shown in fig. 3, the precipitation begins, first, in the neighborhood of $x = -0.108\lambda$ and therefore is distributed along a strip of the ground surface, whose length is 0.358λ, or 8,600 meters; from this we compute the average precipitation per hour, as follows:

$$\frac{3.6 \times 3{,}460}{8{,}600} = 1.45 \text{ mm.}$$

Similarly, we find for winter:

$$G_{x=0} = 380, \quad G_{x=\frac{\lambda}{6}} = 770, \quad G_{x=\frac{\lambda}{4}} = 1{,}264;$$

the total precipitation is distributed over a strip 9,400 meters long, so that the average precipitation is 0.485 millimeters per hour.

From the above three values of $G(x)$ we can graphically construct the course of this function approximately by considering that the tangent to the curve for G is horizontal at its initial point and when $x = +\lambda/4$.

The tangent to the slope of the curve is found by considering its measure W'. Thus we recognize in our case that the maximum of the precipitation in summer is attained between $x = 0$ and $x = -1$, but in winter between $x = 0$ and $x = +2$ kilometers and amounts to $a \times 2.2$ millimeters, or $a \times 0.75$ millimeters per hour, respectively, for a wind velocity of a meters at some very great altitude; furthermore, after passing the summit of the mountain the precipitation diminishes more slowly than was found under our previous assumption of a constant thickness of clouds. In reality, on account of the conveying of the water or ice with the cloud, which we still neglect as before, the maximum of precipitation is pushed still more toward the summit of the mountain. Moreover, since one part of the cloud floats over the summit and is there dissipated in the sinking or descending currents of air, the precipitation will stretch a little beyond the summit, but its total quantity will be less than the computed.

The results of the preceding analysis, namely, that there exists a zone of maximum precipitation on the windward slope of a mountain and that the inclination of the surface of the earth is more important in determining the quantity of precipitation than is its absolute elevation, is conformed by observations, at least for the higher mountains.[11]

ON THE IONISATION OF ATMOSPHERIC AIR.

By C. T. R. WILSON, M. A., F. R. S., dated February 1, from the proceedings, Royal Society, Vol. LXVIII, pp. 151-161, May 4, 1901.

The present communication contains an account of some of the results of investigations undertaken for the Meteorological Council with the object of throwing light on the phenomena of atmospheric electricity.

In a paper[1] containing an account of the results arrived at during the earlier stages of the investigation I described the behavior of positively and negatively charged ions as nuclei on which water vapor may condense.

The question whether free ions are likely to occur under such conditions as would make these experimental results applicable to the explanation of atmospheric phenomena was left undecided in that paper. My first experiments[2] on condensation phenomena had, it is true, proved that in ordinary dust-free, moist air a very few nuclei are always present requiring, in order that water should condense upon them, exactly the same degree of supersaturation as the nuclei produced in enormously greater numbers by Röntgen rays, and I concluded that they are identical with them in nature and that they are probably ions[3]. While, however, later experiments proved that the nuclei formed by Röntgen or uranium rays can be removed by an electric field and are, therefore, ions; similar experiments made with the nuclei which occur in the absence of ionising radiation led to negative results[4]. In the light of facts brought forward in the present paper I should now feel disposed to attribute the negative character of the results in the latter case to the small number of nuclei present[5].

Subsequently to the publication of the work on the behavior of ions as condensation nuclei, Elster and Geitel showed that an electrified conductor exposed in the open air or in a room lost its charge by leakage through the air, and that the facts concerning this conduction of electricity through the air are most readily explained on the supposition that positively and negatively charged ions are present in the atmosphere. The question where and how these ions are produced remained, however, undetermined; it would, therefore, be incorrect to assume their properties, and in particular their behavior as condensation nuclei to be necessarily identical with those of freshly produced ions; the carriers of the charge might consist of much more considerable aggregates of matter than those attached to the ions with which the condensation experiments had been concerned. Moreover, so long as the source and conditions of production of these ions remained undetermined, one could not assume their presence in the regions of the atmosphere where supersaturation might be expected to occur.

Before going further afield in search of possible sources of ionisation of the atmospheric air, it seemed advisable to make further attempts to determine whether a certain degree of ionisation might not be a normal property of air, in spite of the somewhat ambiguous results given by the condensation experiments to which I have referred.

After much time had been spent in attempts to devise some satisfactory method of obtaining a continuous production of drops from the supersaturated condition, I abandoned the condensation method and resolved to try the purely electrical method of detecting ionisation. Attacked from this side, the problem resolves itself into the question: Does an insulated-charged conductor suspended within a closed vessel containing dust-free air lose its charge otherwise than through its supports when its potential is well below that required to cause luminous discharges?

Several investigators from the time of Coulomb onward have believed that there is a loss of electricity from a charged body suspended in air in a closed vessel in addition to what can be accounted for by leakage through the supports.[6] In recent years, however, the generally accepted view seems to have been that such leakage through the air is to be attributed to the convection of the charge by dust particles.

The experiments were begun in July, 1900, and immediately led to positive results. A summary of the principal conclusions then arrived at was given in a preliminary note "On the leakage of electricity through dust-free air," read before the Cambridge Philosophical Society on November 26. Almost simultaneously a paper by Geitel appeared in the Physikalische Zeitschrift[7] on the same subject, in which identical conclusions were arrived at in spite of great differences in the methods employed.

The following are the results included in the preliminary note, which I read:

1. If a charged conductor be suspended in a vessel containing dust-free air, there is a continual leakage of electricity from the conductor through the air.
2. The leakage takes place in the dark at the same rate as in the diffuse daylight.
3. The rate of leak is the same for positive as for negative charges.
4. The quantity lost per second is the same when the initial potential is 120 volts as when it is 210 volts.
5. The rate of leak is approximately proportional to the pressure.
6. The loss of charge per second is such as would result from the production of about twenty ions of either sign in each cubic centimeter per second in air at atmospheric pressure.

Of these conclusions the first four were also arrived at by Geitel.

As Geitel has pointed out, Matteuci[8] as early as 1850, had arrived at the conclusion that the rate of loss of electricity is independent of the potential. He had also noticed the decrease in the leakage as the pressure lowered.[9]

The volume of air used in my experiments was small, less than 500 cubic centimeters in every case, many of the measure-

[11] See Hann "Klimatologie," Vol. I. p. 298.

[1] Phil. Trans., A., vol. 193, pp. 289-308.

[2] Roy. Soc. Proc., vol. 59, p. 338, 1896.

[3] Camb. Phil. Sec. Proc., vol. 9, p. 337.

[4] Phil. Trans., A., vol. 193, pp. 289-308.

[5] The similar results obtained with nuclei produced in air exposed to ultraviolet light require, however, some other explanation.

[6] Perhaps the most convincing evidence of this is furnished by the experiments of Professor Boys, described in a paper on Quartz as an insulator. Phil. Mag., vol. 28, p. 14, 1889.

[7] Physikalische Zeitschrift, 2 Jahrgang, No. 8, pp. 116-119, published November 24.

[8] Annales de Chim. et de Phys., vol. 28, p. 385, 1850.

[9] This was also observed by Warburg, Annalen der Physik u. Chemie, vol. 145, p. 578, 1872.